CHAPTER 8

SUBIRRIGATION

The purpose of this chapter is to examine factors affecting water movement in a subirrigation system. Methods are presented making certain preliminary design calculations to supplement results obtained from DRAINMOD and improve the efficiency of its application. Examples to demonstrate the use of these methods are presented and discussed.

There are basically two operational procedures for subirrigation systems. The most common procedure is to maintain a constant water level elevation in the tile outlet (Figure 8-1). Water is periodically pumped from a well, stream, or other water supply to replenish water which moves from the drains into the soil to supply ET demands and seepage losses from the system. During dry periods, this procedure results in a water table profile which is more or less in steady state. The drain spacing necessary to satisfy crop ET demands depends on the hydraulic conductivity of the soil, peak ET, or consumptive use, height of the water level in the drain, etc. Methods for determining the drain spacing for steady state operation are discussed in the following section.

Another procedure for operating subirrigation systems is to place a weir in the outlet that extends to near the soil surface and, by pumping for an extended period, raise the water table into the root zone of the profile. Then, pumping is topped and the water table is allowed to fall as water is removed by ET and seepage. Pumping is initiated again when the water table reaches a predetermined depth and the process is repeated. Water table profiles for this unsteady state subirrigation process are process are shown schematically in Figure 8-2. Determination of design parameters, such as drain spacing in this situation depends on the time required to raise the water table to the desired elevation. Methods for predicting the time required to raise the water table in terms of drain spacing, hydraulic conductivity, and other factors are given in a subsequent section of this chapter.

## Steady State Operation

The position and shape of the water table for steady-state subirrigation can be approximated by making the Dupuit-Forchheimer (D-F) assumptions and using the approach of Fox, et al, (1956). By neglecting water movement in the unsaturated zone, the flow rate in the horizontal direction per unit length of drain may be expressed as:

$$Q_x = - K h \frac{dh}{dx} \qquad (8\text{-}1)$$

Where, referring to Figure 8-1, $Q_x$ is the horizontal flow rate ($cm^3$/hr cm) and h is the height of the water table above the impermeable layer which depends on the horizontal position, x, (i.e., h = h(x)). At any position, x, $Q_x$ must be equal to the rate that water leaves the profile by ET in the section x to x = L/2. That is,

$$Q_x = e(L/2 - x) \tag{8-2}$$

Then,

$$- K h \frac{dh}{dx} = e \ (L/2 - x) \tag{8-3}$$

Separating variables and integrating subject to the boundary condition of $h = h_o$ at x = 0 yields an expression for the water table position in terms of x:

$$h^2 = \frac{e}{K} x^2 - \frac{e\ L}{K} x + h_o^2 \tag{8-4}$$

Thus, the water table assumes an elliptical shape under steady ET conditions. The derivation of Equation 8-4 assumes that water can move vertically from the water table by unsaturated flow to supply the ET demand. The maximum upward rate of water movement is dependent on water table depth as well as soil properties as discussed in Chapter 2. Therefore, the drains should be placed close enough together to maintain some minimum water table elevation at the midpoint (x = L/2) during a period of maximum ET demand. This spacing can be estimated from Equation 8-4 by specifying a water table elevation of $h_1$ at x - L/2 and solving for L:

$$L = [4\ K(h_o^2 - h_1^2)/e]^{1/2} \tag{8-5}$$

The effective horizontal hydraulic conductivity should be used for K in Equation 8-5, while the maximum permissible water table elevation at the drains, $h_o$, will depend on the root zone depth, crop sensitivity and site parameters.

As discussed above, Equations 8-2 to 8-5 are subject to the D-F assumptions and do not consider convergence losses near the drain. These losses can be accounted for by substituting an effective depth to the impermeable layer, $d_e$, for d in Figure 8-1, as discussed in Chapter 2 (pages 2-13 to 2-15) for drainage. The h values are adjusted accordingly. The value of $d_e$ can be computed from Equations 2-13 and 2-14. Because $d_e$ depends on the drain spacing, L, an iteration process is required to compute L from Equation 8-5. First, a trial value of L is calculated from Equation 8-5 using h values based on the actual value of d. Then, $d_e$ is computed from Equation 2-13 or Equation 2-15 and the $h_o$ and $h_1$ are adjusted. Then, a new value of L is determined from Equation 8-5. A new value of $d_e$ is computed and the process is repeated until L remains constant. Usually, one iteration is sufficient for convergence.

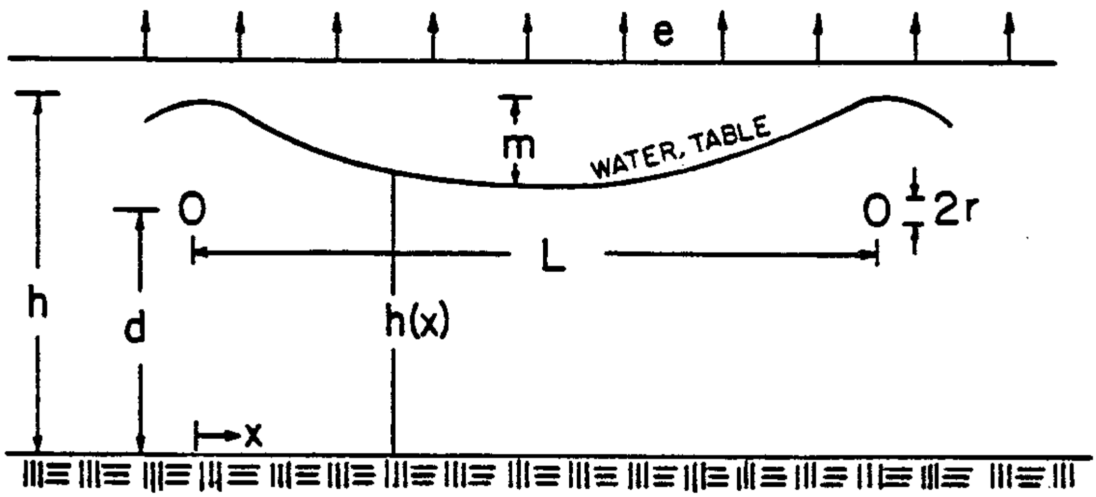


Figure 8-1. Water table profile for subirrigation under steady state conditions with an ET rate of e.

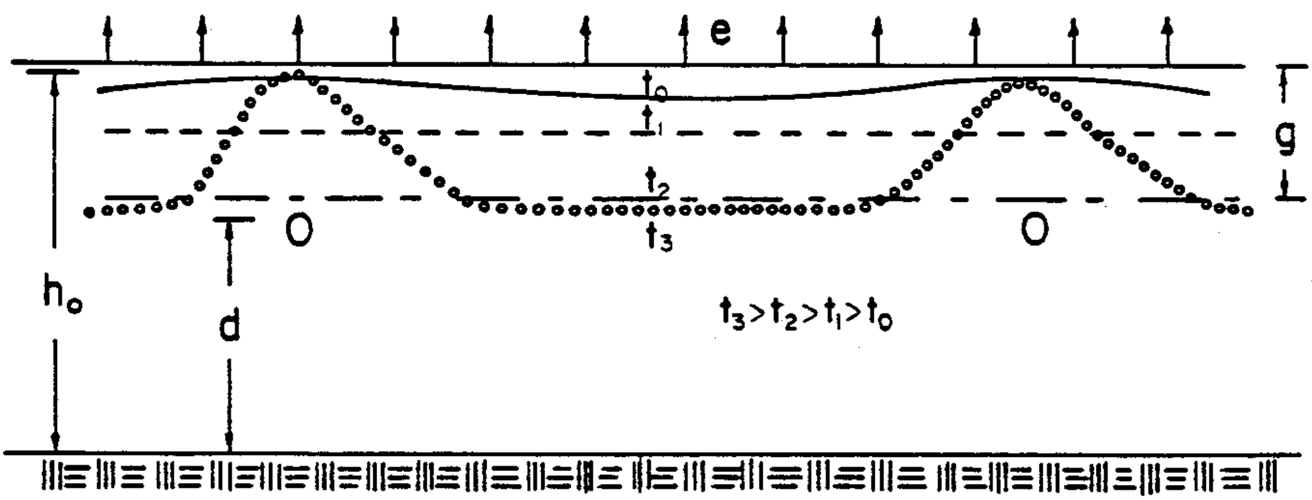


Figure 8-2. Water table profiles for unsteady state operation of a subirrigation system. The water table is raised to near the surface at time, $t_o$. Then, pumping is stopped and the water table recedes due to ET, as shown for times $t_1$ and $t_2$. When the water table reaches some depth, g, pumping is initiated to raise the water table back to its initial position.

Example 1 - Steady State Subirrigation

A Portsmouth sandy loam has a hydraulic conductivity of 3 cm/hr and a profile depth to a restrictive layer of 2.0 m. Drains are placed at a 1 m depth as shown in Figure 8-3 with the main in the direction of the surface slope of 0.5 percent. Corn is to be grown with an effective rooting depth of 30 cm (1 ft.). Roots cannot penetrate much below this depth because of acid subsoil. The drains to be used have a diameter of 10 cm (4 inches) with a completely open effective radius of 0.51 cm. Determine the drain spacing necessary for subirrigation during dry periods in the summer when the peak ET demand is 0.5 cm/day.

Because the root zone is 30 cm deep, the water level in the laterals should not be held closer than 30 cm to the surface. A given depth in the lateral can be maintained in a sloping situation by placing a water level control structure such as those shown in Figure 8-4 immediately below each lateral. One design of such structures is described in detail in an SCS technical note (TECH NOTE ENG-FL-11) from the SCS Florida State Office (dated April 1977). Depending on the slope, it may be possible to service several laterals with a single control structure (Figure 8-3). However, in this case, we will assume that the water level is controlled exactly 30 cm from the surface in each lateral so that $h_o = 100 - 30 + d_e$. Assuming $d_e = d = 100$ cm for the first trial, gives $h_o = 170$ cm. To determine $h_1$, we use the curve in Figure 5-6 for Portsmouth. It gives a water table depth below the root zone of 46 cm for a steady upward flux of 0.5 cm/day. The root zone is 30 cm deep so $h_1 = d + 100 - (30 + 46) = 100 + 24 = 124$ cm. Applying Equation 8-5 gives a first estimate for the drain spacing of:

$$L_1 = [4 \times 3 \text{ cm/hr } (170^2 \text{ cm}^2 - 124 \text{ cm}^2)/(0.5 \text{ cm/day} \cdot \frac{1 \text{ day}}{24 \text{ hr}})]^{1/2}$$

$$L_1 = 27.9 \text{ m (91 ft)}$$

The equivalent depth to the impermeable layer is then calculated using Equation 2-18 with $r = r_e = 0.51$ cm as:

$$d_e = \frac{100}{1 + \frac{100}{2{,}790} \{\frac{8}{\pi} \ln \frac{100}{0.51} - 3.4\}} = 74 \text{ cm}$$

With this value of $d_e$, $h_o = 74 + 70 = 144$ and $h_1 = 74 + 24 = 98$. Then,

$$L_2 = [4 \times 3 \ (144^2 - 98^2)/(0.5/24)] = 25.3 \text{ m} \quad (83 \text{ ft})$$

Recalculating $d_e$ from Equation 2-18 gives $d_e = 72$ cm which is close enough to the 74 cm assumed in the above calculation of L. Therefore, a drain spacing of $L = L_2 = 25.3$ m (83 ft) would be sufficient to supply an ET rate of 0.5 cm/day, if the water level in the drain is held 30 cm from the surface.

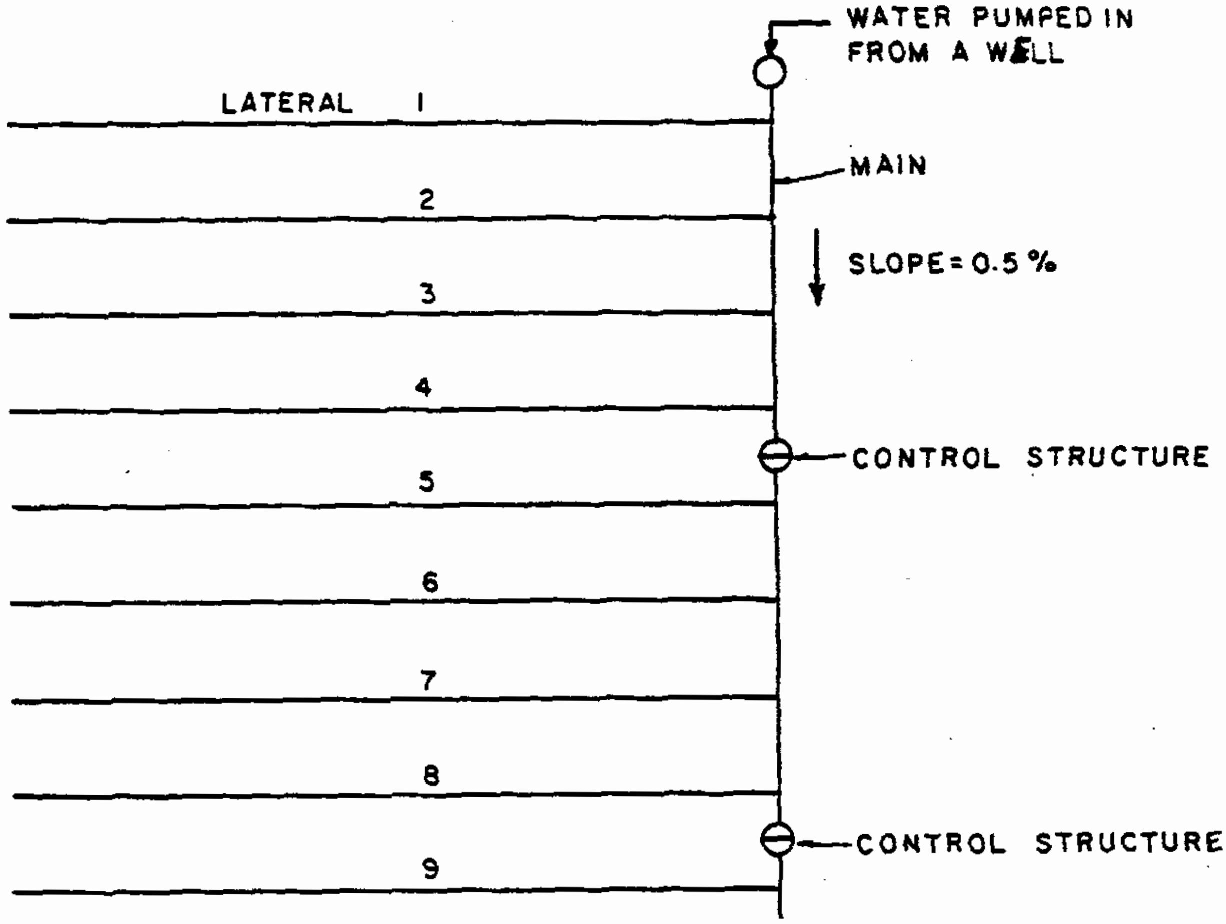


Figure 8-3. Layout of laterals and main with water level control structures.

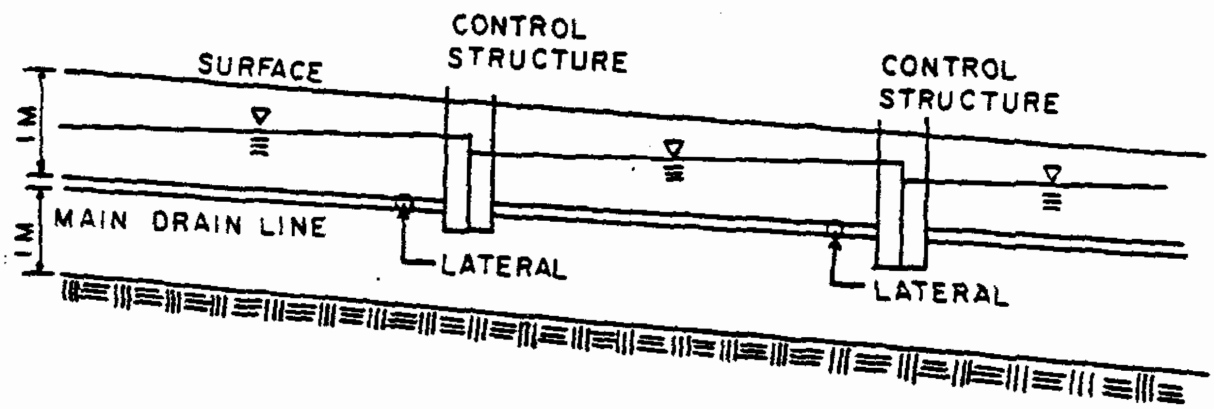


Figure 8-4. Profile view of main drain line with water within a given distance of the surface at the drain lines.

What if the minimum tolerable water table depth is 50 cm, rather than 30, as assumed above? Then, starting with an assumed $d_e$ of 70 cm, we would have $h_o = 70 + (100 - 50) = 120$ cm and $h_1 = 70 + (100 - 30 - 46) = 94$ cm. From Equation 8-5, $L_1 = 17.9$ m. Recalculating $d_e$ gives $d_e = 64$ cm so $h_o = 64 + 50 = 114$ and $h_1 = 64 + 24 = 88$. Then, $L_2 = 17.4$ m and the new $d_e$ is $d_e = 63$ cm, which is close to the assumed value of 64 cm. Therefore, if the water level in the drain line is maintained at a depth of 50 cm, a drain spacing of L = 17.4 (57 ft) would be needed, as opposed to the 25 m spacing for a 30 cm depth.

Water Table Rise During Subirrigation

The time required to raise the water table to a height sufficient to supply crop ET demands may be the limiting factor in the design of a subirrigation system. The need to consider this aspect is obvious for operations where the water table is raised to the root zone and then allowed to fall as water is removed from the profile by ET. These systems function in an unsteady state mode and it is extremely important to be able to raise the water table rapidly enough to maintain a supply of water to the crop. The time required to raise the water table is also important for steady state operation. Ignoring this aspect of the operation could result in a prohibitive length of time to raise the water table at the beginning of the growing season or when irrigation is initiated.

Methods for predicting water table rise for both initially horizontal and draining profiles were presented in a previous paper (Skaggs, 1973). The methods are described here and new graphical solutions are presented for the convenience of the user.

Equation 8-1 for horizontal flow rate may be combined with the principle of conservation of mass to obtain the following governing equation for unsteady conditions (van Schilfgaarde, 1974).

$$f \frac{\partial h}{\partial t} = K \frac{\partial}{\partial x} [h \frac{\partial h}{\partial x}] + e \qquad (8-6)$$

Where, referring to Figures 8-1 and 8-2, $h = h(x,t)$ is the distance of the water table above the impermeable layer, t is time, f is effective or fillable porosity, and e is the rate water is added to the soil by rainfall and is negative for losses by ET or deep seepage. If the water table is initially flat at some distance, $h_i$ above the impermeable layer, the boundary and initial conditions may be written as:

$$h = h_o , \quad x = 0 , \quad t > 0 \qquad (8-7a)$$

$$h = h_o , \quad x = L , \quad t > 0 \qquad (8-7b)$$

$$h = h_i , \quad 0 \le x \le L, \quad t = 0 \qquad (8-7c)$$

Equation 8-6 can be expressed in nondimensional form as:

$$\frac{\partial H}{\partial \tau} = \frac{\partial}{\partial \xi} (H \frac{\partial H}{\partial \xi}) + \mu \qquad (8-8)$$

Where $H = h/h_o$, $\xi = x/L$, $\mu = eL^2/Kh^2$, and $\tau = \frac{K\,h_o}{f\,L^2}\,t$. Then, the boundary conditions may be written,

$$H = 1,\ \xi = 0,\ \tau > 0 \qquad (8\text{-}9a)$$

$$H = 1,\ \xi = 1,\ \tau > 0 \qquad (8\text{-}9b)$$

$$H = D = h_i/h_o,\ 0 \leq \xi \leq,\ \tau = 0 \qquad (8\text{-}9c)$$

The D-F assumptions are not valid for regions near the drain tube, as discussed earlier, so $d_e$ should be substituted for d in Figures 8-1 and 8-2. The values of $h_o$ and $h_i$ should be adjusted accordingly to compensate for convergence losses near the drain.

Solutions

Numerical solutions to Equation 8-8 were obtained by writing the equation in finite difference form and solving on the digital computer. The numerical methods are described elsewhere (Skaggs, 1975). Solutions for the H vs. $\tau$ are given for a point midway between the drain ($\xi = x/L = 0.5$) in Figures 8-5 through 8-8 for $\mu$ values of 0, -1, -2, and -3, respectively. The solutions in each figure are plotted for a range of $D = h_1/h_o$ values from D = 0.0 to D - 0.95. Solutions for D and $\mu$ values not given can be obtained by interpolation.

The final or steady state values of H are constant for a given $\mu$ value, as shown in Figures 8-5 through 8-8. The steady state value of H can be obtained by solving Equation 8-8 with $\partial H/\partial \tau = 0$. Then,

$$\frac{\partial}{\partial \xi}\left(H\,\frac{\partial H}{\partial \xi}\right) + \mu = 0 \qquad (8\text{-}10)$$

Separating variables and integrating subject to the boundary conditions:

$$\partial H/\partial \xi = 0 \text{ at } \xi = 1/2 \qquad (8\text{-}11a)$$

and

$$H = 1 \text{ at } \xi = 0 \qquad (8\text{-}11b)$$

gives

$$H^2 = -\mu\xi^2 + \mu\xi + 1 \qquad (8\text{-}12)$$

At the midpoint, $\xi = 1/2$ and $H_m^2 = \mu/4 + 1$ (8-13)

Then, for example, if $\mu = -1$, the midplane H value should approach $H_m$ = 0.87 after some period of time. This is consistent with results given in Figure 8-6, which shows that the steady state position of H = 0.87 is attained at $\tau = 0.8$ for all D values. Note that for $\mu = -4$, $H_m = 0$

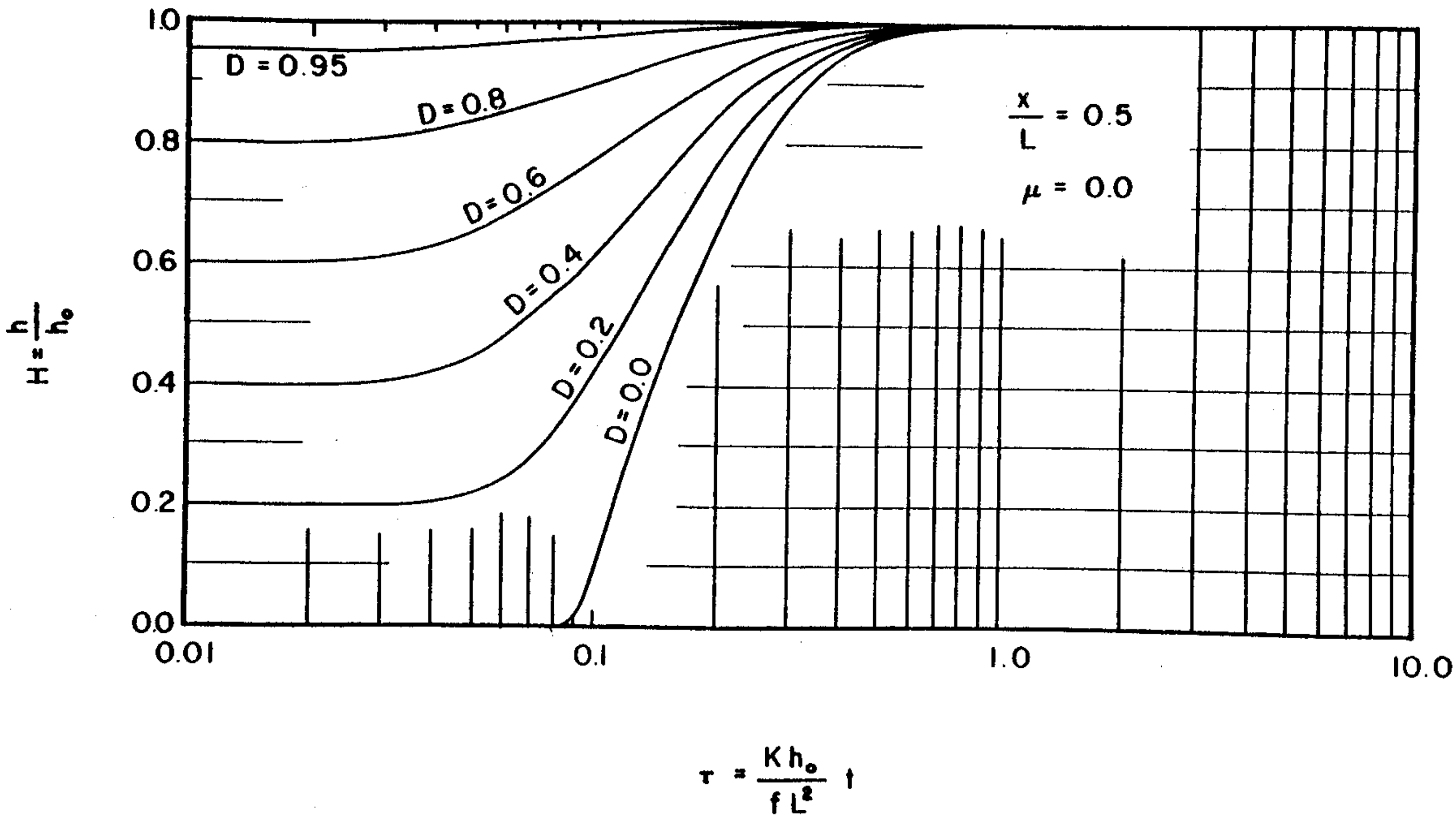


Figure 8-5. Solutions for water table movement at a point midway between the drains when the water table elevation is raised to $h_o$ in the drains. The initial water table is horizontal at an elevation of $h_i$ and $D = h_i/h_o$. The nondimensional vertical loss rate is $\mu = 0$.

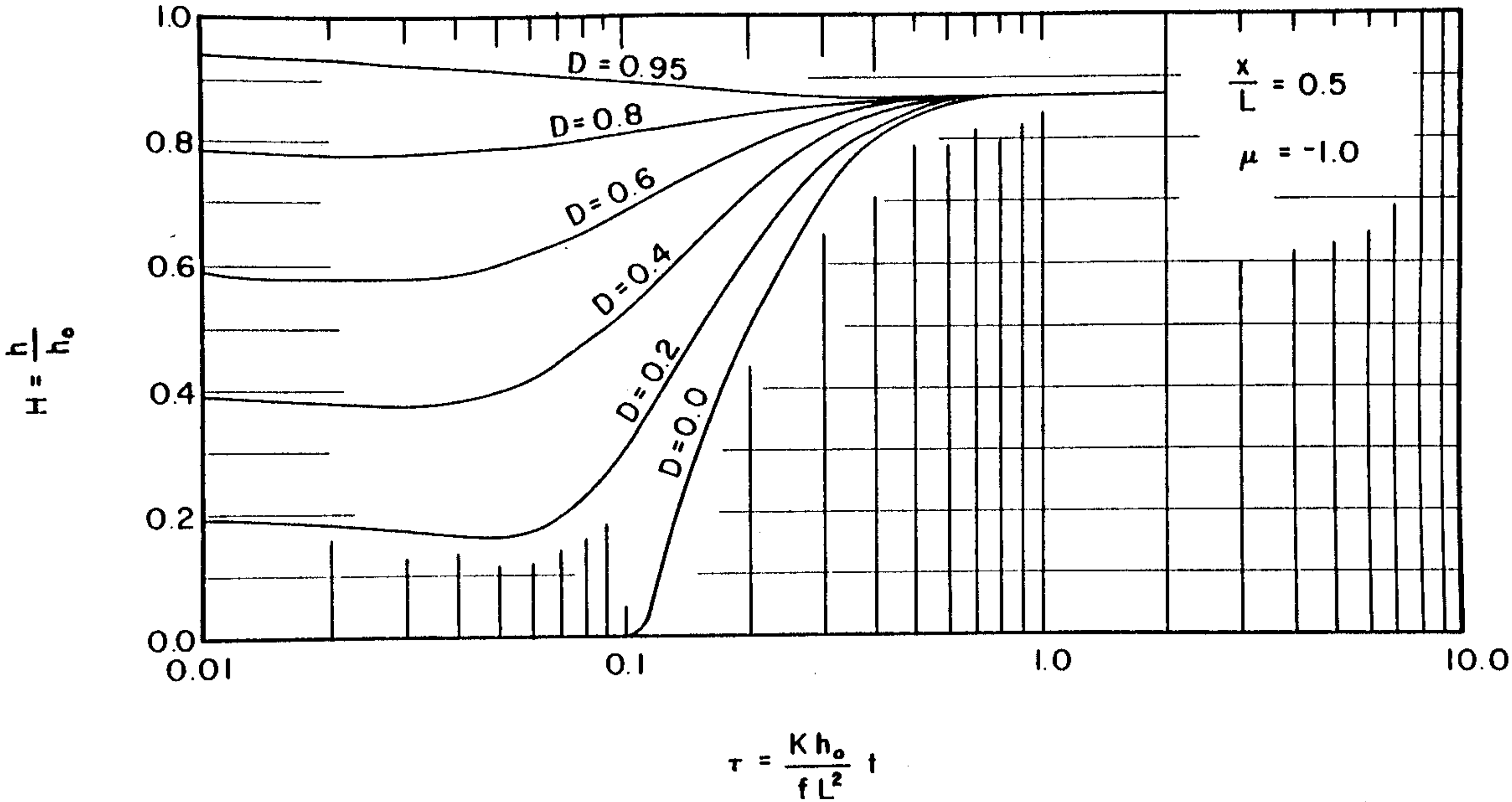


Figure 8-6. Solutions for water table movement at a point midway between the drains when the water table elevation is raised to $h_o$ in the drains. The initial water table is horizontal at an elevation of $h_i$ and $D = h_i/h_o$. The nondimensional vertical loss rate is $\mu = -1.0$.

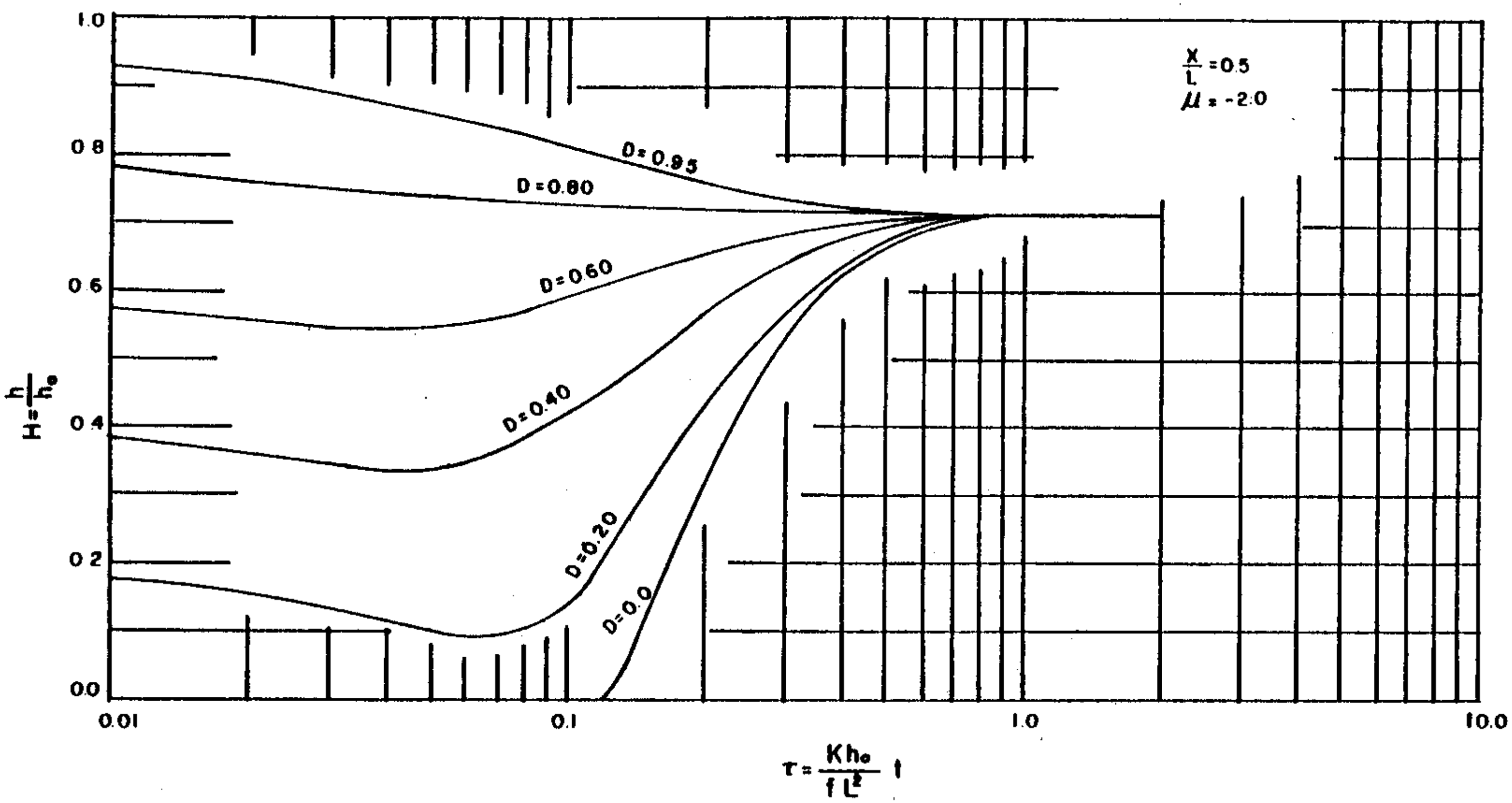


Figure 8-7. Solutions for water table movement at a point midway between the drains when the water table elevation is raised to $h_o$ in the drains. The initial water table is horizontal at an elevation of $h_i$ and $D = h_i/h_o$. The nondimensional vertical loss rate is $\mu = -2.0$.

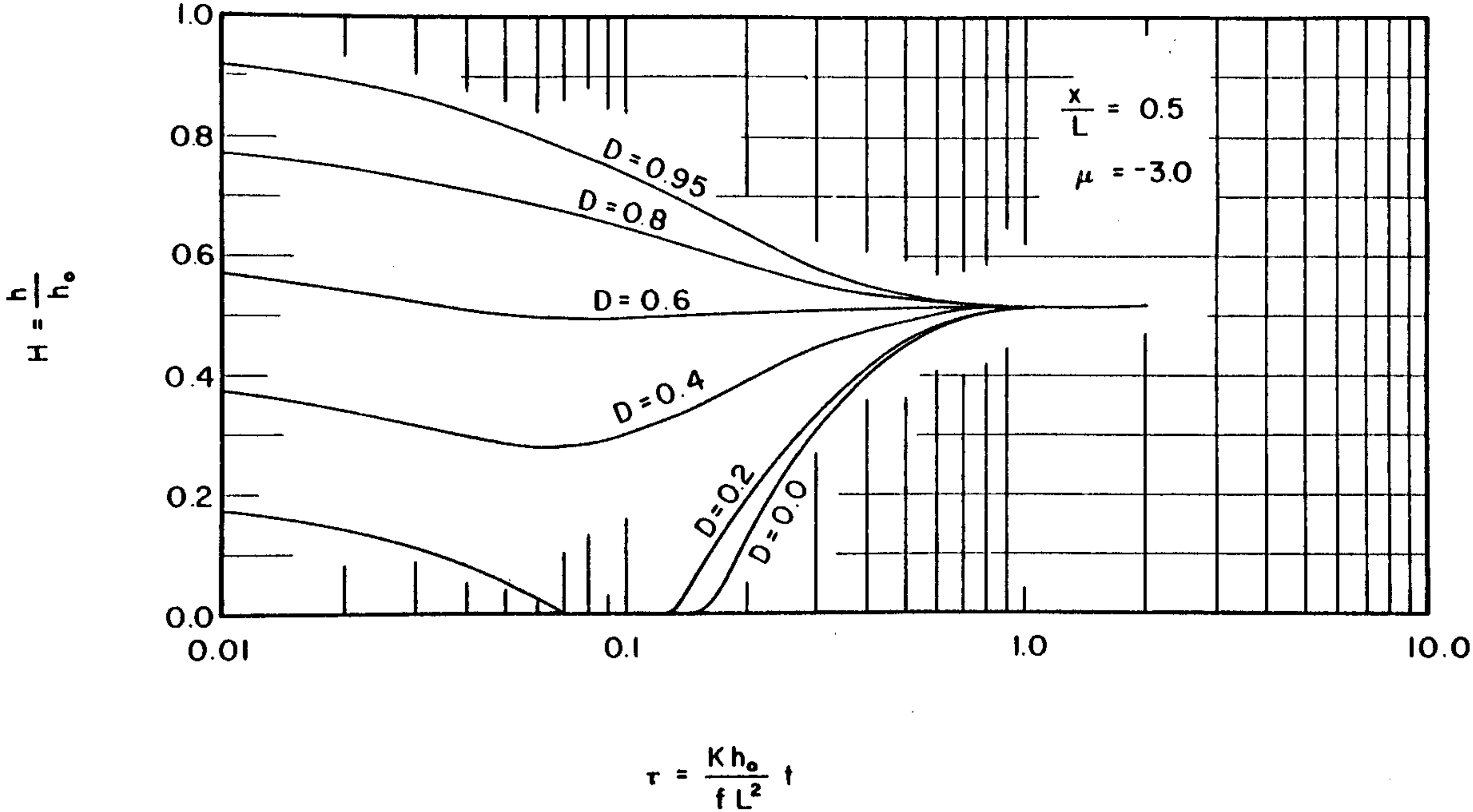


Figure 8-8. Solutions for water table movement at a point midway between the drains when the water table elevation is raised to $h_o$ in the drains. The initial water table is horizontal at an elevation of $h_i$ and $D = h_i/h_o$. The nondimensional vertical loss rate is $\mu = -3.0$.

(Equation 8-13). This simply means that the water table elevation at the midpoint will be drawn down to the impermeable layer by the ET losses when $\mu = eL^2/Kh_o^2 = -4$. This assumes, of course, that the ET rate of e occurs uniformly across the field and is not restricted by the deep water table. In fact, it may be restricted, but this would represent a point of failure for the subirrigation system. In any case, solutions for $\mu < -4$ are not needed as it is not possible to maintain a steady state midplane water table above the impermeable layer for these values.

It may seem unusual that the midplane water table decreases after the water level is raised in the drains (e.g., the solution for D = 0.8, $\mu = -3$ in Figure 8-8). This can occur when the initial water table is higher than the steady state water table depth; i.e., $D > H_m$. In other cases, the midplane water table may decrease for a while then increase (e.g., the solutions for D = 0.4 and 0.2 in Figure 8-8). This happens because some time is required for the water table midway between the drains to react to a change in the water level at the drains. However, vertical losses due to ET (and deep seepage, if it occurs), have an immediate effect. So the midplane water table may fall at first due to ET losses, then increase as water arrives from the drain.

Example 2 - Water Table Rise During Startup

The water table in Example 1 is initially horizontal at a depth of 1 m when the crop is planted and the water level in the drain is raised to within 30 cm of the surface. If the drain spacing is 25 m (from Example 1) and the evaporation rate is assumed to be zero during the period just after planting, how much time will be required to raise the midpoint water table to the design elevation of 76 cm from the surface?

Since e = 0, $\mu = 0$, and Figure 8-5 can be used to calculate the time required. From calculations in Example 1, $d_e = 72$ cm for L = 25 m, so $h_o = 72 + (100 - 30) = 142$ cm, $h_i = 72/142 = 0.51$. The water table at the midpoint is to be raised to $h_1 = 72 + (100 - 76) = 96$ cm. Then, $H = h_1/h_o = 96/142 = 0.676$. The effective porosity for Portsmouth s.l. can be estimated from the slope of the drainage volume - water table depth curve given in Figure 5-4. The slope between water table depths of 1.0 m and 0.75 m is f = 0.06. Substituting H = 0.68 in Figure 5-5 and interpolating for D = 0.51 gives $\tau = 0.089$. Then,

$$\tau = \frac{K\,h_o}{f\,l^2}\;t = 0.089$$

$$t = \frac{0.089\,f\,L^2}{K\,h_o} = \frac{0.089 \times 0.06 \times 2500^2\ \text{cm}^2}{3\ \text{cm/hr} \times 142} = \underline{78\ \text{hours}}$$

Thus, 78 hours will be required to raise the water table to the design elevation, if evaporation from the surface is negligible.

What time will be required for the same situation if the ET rate is a relatively modest 0.20 cm/day? For this case, $\mu = -eL^2/K\,h_o^2 = -0.20$ cm/d x $2500^2$ $cm^2$/(3 cm/hr x $142^2$ $cm^2$ x 24 hr/day) $\mu = -0.86$. Substituting H = 0.68 in Figure 5-6 ($\mu = -1$) gives $\tau_{-1} = 0.137$ and from above $\tau_o = 0.089$. Interpolation for $\mu = -0.86$ yield $\tau_{-0.86} = 0.130$. Solving for t, as shown above, yields:

$$t = \frac{0.130 \times 0.06 \times 2500^2}{3 \times 142} = \underline{114 \text{ hours}}$$

This example shows that a substantial length of time may be required to raise the water table, especially when water is lost by ET from the surface. The time increases sharply with e, as shown in FIgure 8-10, for L = 25 m. The 25 m spacing was determined from steady state considerations in Example 1 such that a water table depth of 76 cm at a point midway between the drains would result if the water level in the drains is held at an elevation 30 cm from the surface and the steady ET = 0.5 cm/day. However, the above results and those given in Figure 8-10, show that a long time would be required to raise the water table to the desired steady state position. For example, if the water table is allowed to drop to a depth of 100 cm for some reason (equipment failure, operator error, assumption that it is going to be a wet year and irrigation will not be needed), about 240 hours would be required to raise the water table to its steady state position, if e = 0.4 cm/day. The irrigation requirement would not be met during that period and substantial yield reductions could result. Therefore, a smaller drain spacing than calculated from the steady state analysis may be desirable to reduce the time required to raise the water table during the growing season.

The time required to raise the midplane water table, as affected by the vertical loss rate, e, is also plotted for L = 17.4 m in Figure 8-10. Only 57 hours would be required to raise the water table for this spacing when e = 0.4 cm/day. Then, the water level at the drains could be allowed to fall to a depth of 50 cm and still supply a steady ET rate of e = 0.5 cm/day (Example 1). This would allow a smaller variation in the steady state water table depth (from 50 cm at the drain, to a depth of 76 cm at the midplane). At the same time, the smaller spacing would provide system that is responsive to adjustments in the outlet water level during the growing season.

The effects of rainfall and of available water stored in the unsaturated zone are not considered in this chapter. The effects of such factors on drain spacing and operational procedures of a subirrigation system can be analyzed best by using DRAINMOD to simulate the performance of the system. However, the methods discussed herein can be used to make a first cut design of the subirrigation system. The methods may also be used to check the final design for the time required to raise the water table to an operational position. Interruptions of subirrigation due to equipment breakdowns or other problems, are not planned so they are not usually simulated when DRAINMOD is used to analyze a given design. Thus, the time required to "restart" the subirrigation process should be checked for all systems designs.

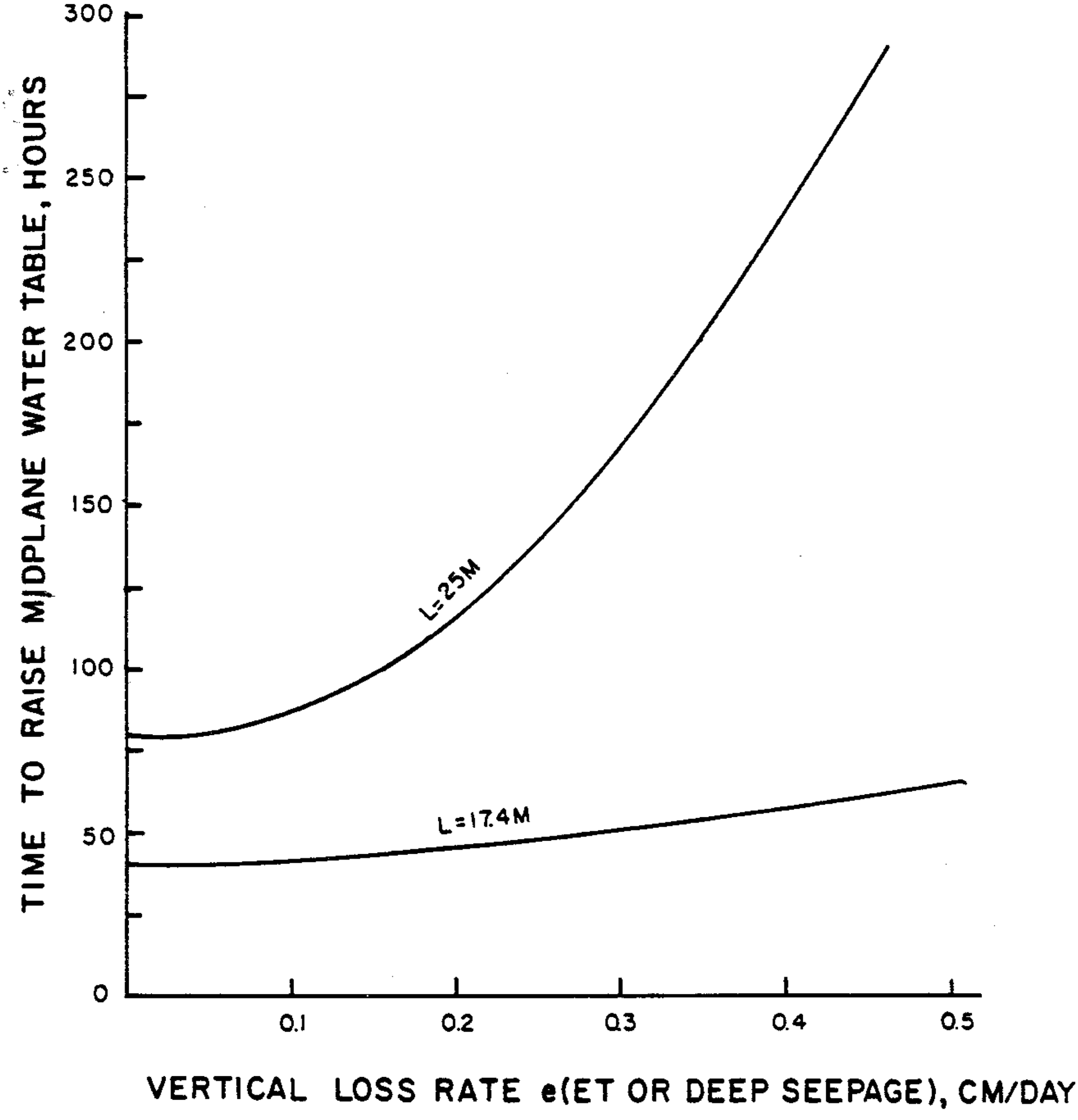


Figure 8-9. Effect of vertical loss rate e on time to raise the midplane water table from a depth of 100 cm to 76 cm for two drain spacings in a Portsmouth s.l. soil. The water level in the drains is raised to within 30 cm of the surface for both drain spacings.